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14. ABSTRACT
The major goals of this project include three components:
First, we want to characterize the optimal number of measurements that are needed to identify any problematic component of a connected complex system. Second, we aim to provide constructive methods to generate those measurements. Finally, we would like to test these theoretical results by numerical simulations. All three goals have been essentially met. Please see the report for more details.

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Final Technical Report for "Algorithmic Approach for Network Inference and Monitoring: Coding over Real Numbers" (N00014-11-1-0131)

PI: Ao Kevin Tang
Institution: Cornell University

Abstract:

The major goals of this project include three components:

First, we want to characterize the optimal number of measurements that are needed to identify any problematic component of a connected complex system. Second, we aim to provide constructive methods to generate those measurements. Finally, we would like to test these theoretical results by numerical simulations.

All three goals have been essentially met. More concretely, for goal number 1, we have shown the lower bound of number of measurements for any given graph. In particular, for several special graphs, we were able to find the exact optimal number of measurements. Furthermore, for general networks, we were able to provide order optimal estimation of the number of measurements needed. For goal number 2, all above estimation comes with explicit measurement construction. In particular, for general networks, we have a construction which only uses polynomial time algorithms. Finally, for goal number 3, all above results are extensively tested numerically.

1. Overall Goals and Achievements

The major goals of this project include three components:

First, we want to characterize the optimal number of measurements that are needed to identify any problematic component of a connected complex system. Second, we aim to provide constructive methods to generate those measurements. Finally, we would like to test these theoretical results by numerical simulations.

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2. Description of Topics and Results

Uniqueness of solution to underdetermined system

We investigate the uniqueness of a nonnegative vector solution and the uniqueness of a positive semidefinite matrix solution to underdetermined linear systems. A vector solution is the unique solution to an underdetermined linear system only if the measurement matrix has a row-span intersecting the positive orthant. Focusing on two types of binary measurement matrices, Bernoulli 0–1 matrices and adjacency matrices of general expander graphs, we show that, in both cases, the support size of a unique nonnegative solution can grow linearly with the problem dimension. We also provide closed-form characterizations of the ratio of this support size to the signal dimension. For the matrix case, we show that under a necessary and sufficient condition for the linear compressed observations operator, there will be a unique positive semidefinite matrix solution to the compressed linear observations. We further show that a randomly generated Gaussian linear compressed observations operator will satisfy this condition with overwhelming high probability.

Sparse Recovery over Graphs

Motivated by network inference and tomography applications, we study the problem of compressive sensing for sparse signal vectors over graphs. In particular, we are interested in recovering sparse vectors representing the properties of the edges from a graph. Unlike existing compressive sensing results, the collective additive measurements we are allowed to take must follow connected paths over the underlying graph. For a sufficiently connected graph with n nodes, it is shown that, using $O(k \log(n))$ path measurements, we are able to recover any k-sparse link vector (with no more than k nonzero elements), even though the measurements have to follow the graph path constraints. We further show that the computationally efficient L_1 minimization can provide theoretical guarantees for inferring such k-sparse vectors with $O(k \log(n))$ path measurements from the graph.

We also the problem of sparse recovery with graph constraints in the sense that we can take additive measurements over nodes only if they induce a connected subgraph. We provide explicit measurement constructions for several special graphs. A general measurement construction algorithm is also proposed and evaluated. For any given graph G with n nodes, we derive order optimal upper bounds of the minimum number of measurements needed to recover any k-sparse vector over G (M^G_{k,n}). Our study suggests that M^G_{k,n} may serve as a graph connectivity metric.

Sparse Error Correction from Nonlinear Measurements

We consider the problem of sparse error correction from general nonlinear measurements, which has applications in state estimation of electrical power networks, when bad data (outliers) are present. An iterative mixed L-1 and L-2 convex program is used to estimate the true state by locally linearizing the nonlinear measurements. In the special case when the measurements are linear, through using the almost Euclidean property for a linear subspace, we derive a new performance bound for the state estimation error under sparse bad data and additive observation noise. As a byproduct, in this paper we provide sharp bounds on the almost Euclidean property of a linear subspace, using the "escape-through-the-mesh" theorem from geometric functional analysis. When the measurements are nonlinear, we give conditions under which the solution of the iterative algorithm converges to the true state even though the locally linearized measurements may not be the actual nonlinear measurements. We are able to use a semidefinite program to verify the conditions for convergence of the proposed iterative sparse recovery algorithms from nonlinear measurements. We then numerically evaluate our iterative convex programming approach of performing bad data detections in nonlinear electrical power networks problems.

3. Student Training

Several results from this project have been included in a graduate course (ECE 5800: Control and Optimization of Information Networks) at Cornell. The grant also partially supported two PhD students (Meng Wang and Enrique Mallada). Meng graduated in 2012 and is a now a tenure-track assistant professor at RPI. Enrique graduated in 2013 and is now a CMI postdoc at Caltech.

4. Results Dissemination

In terms of publications, we have published most related results. Conference papers include on in 2011 Assilomar conference, three in 2011 ISIT, two in 2011 CDC, one in 2011 Infocom and one in 2012 Infocom. We also published several journal papers including two in IEEE Trans on Signal processing, one in IEEE Transactions on Information theory.

In terms of talks, besides several conference presentations, we have also given several invited seminars at various places including Caltech, MIT, Princeton, Northwestern and USC.

5. Future Direction

One of my future directions is to further study Tomography problems and its applications in networking. Right now, the network topology is assumed to be given and the goal is to recovery a snapshot of network states (static formulation). We plan to further examine methods to recover underlying topology and to include dynamics in the framework of sparse recovery. I also plan to focus more on nonlinearity in sparse recovery problems.

6. Main Related Publication

[1] M. Wang, W. Xu and A. Tang "A Unique "Non-negative" Solution to an Underdetermined System: from Vectors to Matrices", *IEEE Transaction on Signal Processing*, 59(3): 1007-1016, March 2011.

[2] M. Wang W. Xu and A. Tang, "On the Performance of Sparse Recovery via Lp-minimization (0<=p<=1)", *IEEE Transaction on Information Theory,* 57(11):7255-7278, November 2011.

[3] W. Xu, E. Mallada and A. Tang, "Compressive Sensing over Graphs", Proceedings of Infocom 2011.

[4] W. Xu and A. Tang, "On the Scaling Law for Sparse Signal Recovery and its Applications", Proceedings of CISS, 2011.

[5] W. Xu, Meng Wang, E. Mallada and A. Tang, “Recent Results on Sparse Recovery over Networks”, (invited) Proceedings of Asilomar Conference on Signals, Systems and Computers, 2011.

[6] M. Wang, W. Xu and A. Tang, “On State Estimation with Bad Data Detection”, Proceedings of IEEE CDC 2011.

[7] M. Wang, W. Xu, E. Mallada and A. Tang, “Sparse Recovery with Graph Constraints: Fundamental Limits and Measurement Construction”, Procedings of IEEE Infocom, 2012.

[8] W. Xu, M. Wang, J. Cai and A. Tang, “Sparse Error Correction from Nonlinear Measurements with Applications in Bad Data Detection for Power Networks”, *IEEE Transactions on Signal Processing*, 61(24): 6175-6187, December 2013.

[9] M. Wang, W. Xu, E. Mallada and A. Tang, “Sparse Recovery with Graph Constraints”, submitted to IEEE Transactions on Information Theory, September, 2013